Pattern Recognition Concept in Learning Negative Numbers Subtraction Operation

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***Abstract* -- The aim of this study was to investigate whether pattern recognition concept can be used to help leaners learn to devise a thinking process in solving negative numbers subtraction operation. A survey was conducted involving 30 students aged 11 years old from a primary school in Malaysia. The investigation was segregated into five activities involving three tasks and each task were further segregated into two types. The time taken to conduct this investigation was two and half hours. The findings revealed that for Task 1: Type I and Type II, (90%); Task 2: Type I and Type II, (100%); and Task 3: Type I (100%) and Type II (65%) students were able to devise a thinking process in solving that arrives at a correct answer. This investigation shows that pattern recognition concept can be used to help leaners learn to devise a thinking process in solving negative numbers subtraction operation but further instigation need to be conducted to identify the reasons for such results. Nevertheless, this study integrated computational thinking concept into Mathematics pedagogical aspect and opened a new area for further studies in visualization of information.**

***Keywords* -- Computational Thinking, Pattern Recognition, Visual Informatics, Negative Numbers**

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I. INTRODUCTION

Computational thinking is the new literacy of the 21st Century and a fundamental skill for everyone which enables you to bend computation to your needs and not just for computer scientists (Wing, 2010; Wing, 2006). Wing (2006), further asserted that as to reading, writing, and arithmetic, we should add computational thinking to every child's analytical ability. This investigation was to study if learners comprehend Mathematical concepts integrating computational thinking concept. The computational thinking concept, pattern recognition was adapted. The researchers investigated whether pattern recognition concept enabled leaners learn to devise a thinking process in solving negative numbers subtraction operation.

Literatures show that until today children still struggle with negative number operations (Havita and Cohen, 1995; Borba and Nunes, 2001; Chen and Hung, 2003; Stanford, 2003; Carnellor 2004; Prather and Alibali, 2004; Terao et al., 2005; Brumbaugh & Rock, 2006; Elango and Halimah, 2009; Armstrong, 2010). There are many pedagogical approaches educator have introduce in teaching and learning negative numbers operation. Nevertheless, many students still struggle to adapt its concept cognitively because of its absurd rules (Stanford, 2003). Now, this investigation had opened a new way in addressing integration of computational thinking concept, in particular, pattern recognition concept into teaching and learning solving Negative Numbers subtraction operation.

According to Wing (2010), informally, computational thinking describes the mental activity in formulating a problem to admit a computational solution. In this investigation, the challenge was in designing and developing appropriate shape and arranging it into a pattern which will denote a Mathematical operation concept, in this case, negative numbers subtraction operation. Therefore, the focus was to investigate whether the pattern shape designed and developed can help learners towards learning the concept of solving negative numbers subtraction operation involving two single digits only.

II. LITERATURE REVIEW

According to Prather and Alibali (2004; 2008), their findings suggested that adults' representations of operation with negative numbers were not as well established as their representations of operation with positive numbers. Furthermore, in the operation of negative numbers, some students assume many mathematical things to be universally true (Brumbaugh & Rock 2006). Thus, many times they were amazed to realize their assumptions have been false (Armstrong 2010; Brumbaugh & Rock 2006). For example, some students were unaware that the commutative property for addition operates in sets other than the counting number and a series of questions or problems like $^{-}3 + ^{+}7 =$ and $^{+}7 + ^{-}3 =$ could help lead to the appropriate conclusions and can be amplified with problems involving subtraction where '*commutativity*' does not generally hold. Sometimes, the same students assume to be true ($^{-}5 - ^{+}8 =$ and $^{+}8 - ^{-}5 =$) (Brumbaugh & Rock 2006). Moreover, according to Borba and Nunes (2001), qualitative analysis of their study also showed that the difficulties with having to make explicit the negative numbers involved in the problems could be overcome when children marked the positive and the negative numbers differently; when negative numbers were differentiated from the operation of subtraction; and when children correctly interpreted the results obtained from operating on the explicit representations they had generated. Furthermore, they added that the explicit representation could be either in writing or by the use of manipulative material chosen amongst those made available (coloured cards, marbles, rulers or sticks).

However, Armstrong's (2010) use of a *PowerPoint* presentation involving building sandcastles and digging holes was found to illustrate a direct teaching as indicated in Table 1 and through formative assessment during the lesson. It became apparent that the lower ability students found the model easy to understand where the weakest student in the class, found the lesson particularly accessible and was highly enthused at the pictures and explanations being used as examples when that weakest student typically struggled in mathematics while higher ability students became confused (Armstrong 2010).

TABLE 1: CALCULATION ANALOGY

-1 + 1	A hole plus a sandcastle gives a level surface, since the sand from the sandcastle fills the hole, so answer = 0
3 + -3	3 sandcastles plus 3 holes gives a level surface, so answer = 0
3 – 5	3 sandcastles take away 5 sandcastles, which is the same as flattening 3 sandcastles, then taking away 2 more sandcastles, thus making 2 holes, so answer = - 2
4 + -2	4 sandcastles plus 2 holes. 2 of the sandcastles fit in 2 of the holes, leaving 2 sandcastles remaining, so answer = 2
3 + -5	3 sandcastles plus 5 holes. The 3 sandcastles fill 3 of the holes, leaving 2 holes remaining, so answer = - 2
-2 + -3	2 holes plus another 3 holes. This gives 5 holes in total, so answer = - 5
-7 – - 4	7 holes take away 4 holes. Taking away a hole means filling in the hole with the sand of one sandcastle. Filling in 4 holes leaves 3 holes, so answer = - 3
-2 - - 3	2 holes take away 3 holes. Having taken away (or filled in) 2 holes, we have enough sand to make one sandcastle, so answer = 1

According to a study by Terao et al. (2005), they began by searching major textbooks used in Japan for possible solution methods to the addition and the subtraction problems which incorporated negative numbers. Among the several possible solutions that were found, two of them are described below.

i. The Pictorial Solution (Terao et al. 2005) - This solution required the solver to use a written or mental number line because the calculation was equivalent to movement on a (mental) number line. This solution is an extension of a solution taught in the elementary arithmetic, where adding a positive number was equivalent to going right on the mental number line and subtracting it was equivalent to going left. The elementary-maths version of this solution had been called the counting-on strategy. Based on this solution, the student initially looked at the second number (i.e., b in $a \pm b$) of the expression. If the number was a negative number, the student then tried to change it to a positive number. This would be done by changing the problem $a + b$ to $a - (-b)$ or $a - b$ to $a + (-b)$. For example, if the problem was $2 + (-5)$, the student would change it to $2 - 5$. If the second number was a positive number, the student would do nothing at this stage. Then the student moved right (addition) or left (subtraction) along the number line from the point of the first number, and reached the point of the answer. To ensure that this solution was grasped in detail, and there were no loop holes in the description of the solution, it was implemented with a set of LISP functions.

   A more sophisticated version of this solution does not use a written number line. Students no longer do counting. This sophisticated solution totally depend upon a mental number line and makes use of a part-part-whole relation on a mental number line. For example, consider the problem $2 - 5$. Firstly move from 0 towards the point of the first [add-end] 2 on mental number line. Then one imagines oneself to go 5 to the left. Now the whole length is 5, the length of the right part is 2, and the length of the left part is unknown. If one uses a part-part-whole relation, one will find that the unknown length is 3. The answer to this problem is -3, not +3, because zero is passed when moving from 2, a point in the positive side. The more sophisticated solution with a set of the Locator/Identifier Separation Protocol (LISP) functions was implemented in the current study.

ii. The Algebraic Solution (Terao et al. 2005) - This solution was taught in all textbooks (Terao et al. 2005). The description of the textbooks can be easily translated into a set of production rules shown in Table 2. First of all, one has to confirm the operator is addition. If not, one needs to change the problem to the equivalent addition problem. The production P0 in Table 2 changes the problem from $a - b$ into $a + (-b)$. Then one looks whether the signs of the two add-ends are the same or different. If the signs are in common, one adds the absolute value of the second add-end to the absolute value of the first addend (see production P1 in Table 2), and the polarity of the answer (positive or negative) is consistent with the common sign of the two add-ends (production P4). If the signs are different, one compares the absolute value of the first add-end with the absolute value of the second add-end, subtract the smaller absolute value from the larger absolute value (production P2), and the polarity of the answer is consistent with the sign of the add-end of larger absolute value (production P5).

According to Chen and Hung (2003), to process information and sorting them in a meaningful way is determined by the rules and principles employed. Thus, learning is then perceived as appropriating these rules and principles and being able to apply (or process information) according to these rules. As such, the knowledge of how children construct their early knowledge can be effectively gained from observing and interviewing during explicit teacher set tasks, that is if a student computes that 8 – 5 = 6, and from examination of work samples the teacher would immediately conclude that the child was experiencing difficulties with the subtraction process but with further observation as the child works through the examples: 7-3 = 2; 10 – 7 = 3; 2 – 1 = 4, the teacher quickly realises the source of the errors that is, the child is confusing the digits 2 and 5 (Carnellor 2004). Moreover, a study conducted by Terao et al. (2005) to refine students' skills of addition and subtraction including negative numbers with a seventh grade student, turned out that errors were due to bug rules and the lack of a critical production when executing a purely algebraic solution. These were identified based on a cognitive task analysis using several possible ways of calculation (Terao et al.

2005). Nevertheless, according to Jensen (2008), pattern recognition depends heavily on what experience one brings to a situation and our neural patterns are continually revised as new experiences provide us with additional information, insights and correction. In fact, learning is the extraction of meaningful patterns from confusion (Jensen, 2008). In other words, figuring out in ones' own way and for young children, cognitive understanding is limited by their ability to create personal metaphors or models for information (Jensen 2008).

Different strategies were used by various researchers in helping remedial students gain the knowledge of solving negative numbers subtraction operation (Carnellor 2004; Terao et al. 2005; Armstrong 2010). Nevertheless, Stanford (2003) said that we have been given absurd rules to apply to this weird concept such as: "*a negative number multiplied by a negative number equals a positive number",* and questioned that how can it be that a negative number, which by the definition mathematicians have given us, is less than zero, when multiplied by another number that is less than zero, become a positive number? It has to be pure, unadulterated nonsense and it is clear that the real objects manipulation for the subtraction operation of the negative numbers is an illusion as the negative numbers are imaginary numbers.

TABLE 2: PRODUCTION RULES FOR AN ALGEBRAIC SOLUTION

P0: Change-subtraction-to-addition
If the operator is subtraction then change the operator to addition and change the sign of the second addend.
P1: Do-absolute-addition
If the operator is addition and the two addends have the same sign and no absolute-value addition has been executed then add the absolute value of each addend together.
P2: Do-absolute-subtraction
If the operator is addition; A bug version of this production; lacks this part of the condition. And the two addends have different signs and no absolute-value subtraction has been executed. Then, subtract the smaller absolute value from the larger absolute value of the two digits.
P3: Do-absolute-subtraction-same-sign
If the operator is subtraction and the two digits have the same sign and no absolute-value subtraction has been made then subtract the smaller absolute value from the larger absolute value of the two digits.
P4: Attach-sign-to-common-sign-problem
If the two digits have the same sign and an absolute-value calculation has been executed, then attach the common sign to the result of the absolute calculation.
P5: Attach-sign-to-different-sign-problem
If the two digits have different signs and an absolute-value calculation has been executed, then attach the sign of the digit of larger absolute value to the result of the absolute calculation. This algebraic solution does not require explicit use of a written or mental number line, although we can justify this solution by considering a pictorial solution similar to the sophisticated pictorial solution described in the previous subsection.

Such phenomena is explained by Naylor (2006) as a situation whereby in many parts of the world, students learn a subtraction algorithm different from our own and this algorithm makes a great puzzle for students. Meanwhile, the research conducted by Havita and Cohen (1995) had identified difficulties in students' understanding of the concepts of either signed or negative numbers and in operations on these numbers. Moreover, Brumbaugh and Rock (2006) claimed that it is important for students to determine what things are as well as what they are not, if we are to help them avoid arising at incorrect assumptions, conclusions, thought processes and generalizations. Hence, they suggested that assistance should be provided to the discovery process through a carefully developed set of problems that guide students to appropriate responses (Brumbaugh & Rock 2006). Nevertheless, according to Holmes (2009), teaching is a complex endeavor that requires teachers to meld knowledge about the nature of learners, pedagogical strategies and discipline content.

Bound on the Malaysian education system, negative numbers operations are taught only in the secondary schools starting from the age of 13 years old. Thus, with reference to Table 3, primary developmental stages corresponded to stages that every human moves through while learning suggests the use of abstract symbols and formulas, which would be appropriate for learners in the formal operational stage (Piaget & Inhelder 1969; 1973) in Woolf (2009), clearly indicates that abstraction thinking process in a need to learn negative numbers operation.

TABLE 3: PIAGETIAN STAGES OF GROWTH FOR HUMAN KNOWLEDGE (PIAGET & INHELDER 1969; 1973)

Cognitive Stages	Years	Characterization
Sensorimotor Stage	0 – 2	Motor actions and organizing the senses.
Preoperation Period	3 – 7	Intuitive reasoning without the ability to apply it broadly.
Concrete Operation Stage	8 – 11	Concrete objects are needed to learn logical intelligence.
Formal Operations	12 – 15	Abstract Thinking.

Source: Woolf (2009, pg 114)

A preliminary study conducted by Elango and Halimah (2009) revealed that students have difficulties in solving negative number subtraction operation involving single and double digits. The literature review on the same matter showed that teachers were very creative and innovative in teaching the concept of subtraction and addition operation involving negative numbers. They used various communication tools such as a line graph, coloured stones, coloured chips, gain-owe techniques and computer courseware in their efforts to help students acquire the knowledge of solving negative numbers subtraction and addition operations. These efforts indicated that many types of communicational tools have been used by teachers to help students grasp the concepts of negative number subtraction and addition operations. Furthermore, these efforts also showed the commitment and creativeness of teachers that should be acknowledged as an ongoing process that are continuously evolving in searching ways to help students acquire knowledge and skills related to subtraction and addition operations in negative numbers. Nevertheless, according to Brumbaugh and Rock (2006), it is also important for students to determine what things are as well as what they are not, if teachers are to help them avoid arising at incorrect assumptions, conclusions, thought processes and generalizations.

Furthermore, Carnellor (2004) explains that these difficulties experienced by students will range from misunderstandings within a simple mathematical strand or sub-strand and are usually rectified very quickly to quite severe disabilities that may permeate multiple domains. In order to appropriately guide children's Mathematics learning, she suggests investigating into the relationship of matching, classifying, comparing and ordering (seriation). Meanwhile, Dunn (1994) explained it as a task at hand primarily, to test the feasibility of using more objective procedures in the analysis of students` errors (misconceptions, buggy algorithms) in arithmetic and mathematical operations and the secondary effort used the concepts of skills hierarchy and task analysis to tailor teacher intervention strategies that could be expected to be optimist remedial instruction for students "*at risk*" in some areas of academic performance. Hence, Brumbaugh and Rock (2006) suggested that one of the best ways to have students learn the required skills is to coach the necessary drill in a format that does not appear to focus on the task at hand.

III. METHODOLOGY

A survey was conducted involving 30 students aged 11 years old from a primary class in Malaysia. The Mathematics grade from the recent exam shows that 4 students achieved grade A, 3 students achieved grade B, 15 students achieved grade C and 8 students achieved grade D. It shows that all 30 students like to learn Mathematics. 26 students know what whole number is. Meanwhile, none of them knew what positive or negative numbers were. Moreover, 6 of them have heard about negative numbers and 24 of them had never. All 30 students agreed that they cannot solve negative number operations.

The researchers designed three symbols to help in this investigation as depicted in Figure 1. There are a filled dot, a triangle and an equal sign. Researchers used cardboard to develop these symbols so that hands on activities can be carried out. These three symbols were arranged to create a pattern using these shapes that denotes negative numbers subtraction operation sentence question and answer.

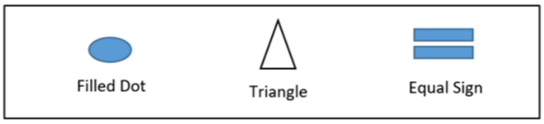

Figure 1. Pattern Shapes Symbols

The data collection was divided into three task. Task 1 consist of sentence questions devised from symbols involving three symbols involving filled dots, triangles and equal sign; Task 2 consist of sentence questions devised from symbols and numbers involving two symbols (filled dots & equal sign) and replacing triangles with numbers; Task 3 consist of sentence questions similar as in Task 2 with an added symbol (filled dot) and number. Therefore, Task 3 is an advancement of Task 2 sentence question. Task 3 was included to see if students were able to assimilate what they have accommodated in Task 2. The Task 1, Task 2 and Task 3 sentence questions are as in Appendix I.

In each Task, there were two types of pattern shapes introduced in this investigation and named as Type I pattern shape and Type II pattern shape. Type I pattern shape: starts with a filled dot and followed by triangles, followed by a filled dot and followed by triangles followed by an equal sign, then the answer will be a filled dot followed by the total number of triangles. For example, a dot followed by one triangle followed by a dot followed by three triangles will give an answer of a dot followed by four triangle as illustrated in Figure 2. Three more examples are as in Table 2.

(●Δ●ΔΔΔ = ●ΔΔΔΔ).

Figure 2. Type I Pattern Shape Sentence Question Equation

Type II pattern shape: starts with triangles followed by a filled dot and followed by triangles (number of the beginning triangles should be less than the number of second triangles after the dot) followed by an equal sign, then the answer will be a filled dot followed by the total number of triangles. For example, one triangle followed by a dot followed by three triangles will give an answer of a dot followed by two triangles as illustrated in Figure 3.Three more examples are as in Table 3.

(Δ●ΔΔΔ = ●ΔΔ).

Figure 3. Type II Pattern Shape Sentence Question Equation

This investigation was segregated into five activities as follows:

Activity I (10 minutes):
In activity I, the researchers explained two examples of pattern recognition concept used in daily life to the students. They were fingerprint recognition and voice recognition concept.

Activity II (10 minutes):
In activity II, the students were divided into 10 groups with three students in each group randomly. Each group was given three filled dot, 10 triangles and one equal sign.

Activity III (40 minutes): Type I Pattern Shape
Activity III was carried out as follows: First, the researchers taught all students Type I pattern shape based on the three examples in Table 4. It was hands on group activity whereby students in each group arranged the pattern shapes given. Then, each group was asked to create Type I pattern shape equation and request other groups to create its output pattern shape and vice versa.

TABLE 4: TYPE I PATTERN SHAPE EXAMPLES

Pattern Shape	Example	Input		Output
Type I	1	●Δ●Δ	=	●ΔΔ
	2	●Δ●ΔΔ	=	●ΔΔΔ
	3	●ΔΔΔ●Δ	=	●ΔΔΔΔ

Activity IV (40 minutes): Type II Pattern Shape

Activity IV was carried out as follows: First, I taught all students Type II pattern shape based on the three examples in Table 5. It was hands on group activity whereby student in each group will arrange the pattern shapes given. Then, each group was asked to create Type II pattern shape equation and request other groups to create its output pattern shape and vice versa. This activity took about 30 minutes.

TABLE 5: TYPE II PATTERN SHAPE EXAMPLES

Pattern Shape	Example	Input		Output
Type II	1	ΔΔ ●ΔΔΔ	=	● Δ
	2	ΔΔ●ΔΔΔΔ	=	● ΔΔ
	3	ΔΔΔ●ΔΔΔΔΔ	=	●ΔΔ

Activity V: Task I, Task II & Task III

After completing all the above activities, each student was requested to complete Task I as in Appendix I individually. Task I was completed in 15 minutes by all students. Then, each student was requested to complete Task II as in Appendix I individually. Task II was completed in 15 minutes by all students. Then, each student was requested to complete Task III as in Appendix I individually. Task III was completed in 20 minutes by all students. The collected data was analyzed descriptively.

IV. FINDING AND DISCUSSION

The integration of pattern recognition concept into Mathematics pedagogical aspect was a challenge as it involves the design and development of pattern using specific shapes to match Mathematical concept that need to be addressed. The Type I pattern shape involves subtraction operation between negative number and positive number. For example -4 - 5 = -9. Meanwhile, Type II pattern shapes involves subtraction operation between positive number and positive number, whereby the first positive number value is less than the second positive number value. For example 3 – 4 = -1.

The findings from Task 1 shows that for pattern shape Type I (item 1, 2, 6 & 7) 27(90%) of students were able to sketch the correct shape pattern output whereas 3 (10%) were unable. Meanwhile, for pattern shape Type II (item 3, 4, 5, & 8) 27 (90%) of students were able to sketch the correct shape pattern output for item 3, 4 & 8 and 18 (60%) of students were able to sketch the correct shape pattern output for item 5. The findings of Task II shows that for both pattern shape Type I and Type II all the students answered correctly. As can be seen, in Task II, I have removed the triangle shape and replace it with whole numbers. The findings from Task II shows that for pattern shape Type I and Type II, 30 (100%) students were able to sketch the correct shape pattern. The findings from Task III shows that for pattern shape Type I (item 1, 3, 4, 8) 30 (100%) students were able to sketch the correct shape pattern whereas for item 2, 5, 6 and 7 which needs the combination knowledge of solving Type I and Type II questions in one sentence question. The findings shows that for item 2 and item 5, 21 (70%) able to sketch the correct shape pattern output. For item 6 and item 7, 18 (60%) able to sketch the correct shape pattern output. Even though the students made some mistake in Task I but in Task II they did not do any mistakes. Whereas, Task III was to see how far can the students adapt and accommodate the thinking they have learned and bring it to an advanced level. Nevertheless, about 76% of students were able to sketch the correct shape pattern output for Task III.

Such phenomenon should be considered as unique. It is because none of the students knew about negative numbers or operation involving negative numbers. It can be confirmed that all the students involved in this investigation have not been formally taught such knowledge during their primary Mathematics education. Therefore, the result shows that students are able to solve negative numbers subtraction operation without knowing that the pattern shapes operation are designed in such a way. This reveals an important point that is even though students struggle to learn Negative Numbers subtraction operation concept that are sometime seems absurd but with such creative pedagogical approach they can learn it. It was interesting and knowledgeable to arrive at such findings.

Therefore, this investigation has proved that pattern recognition concept can be used to help leaners solve negative numbers subtraction operation. Moreover, this investigation also proves that all child can learn Mathematics if given a proper pedagogical guidance that would create more understanding of ways and means how to compute the thinking process to arrive at correct thinking process in teaching and learning Mathematics. Thus, pattern recognition concept, a computational thinking concept, can be used to help leaners towards teaching solving Negative Numbers subtraction operation.

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APPENDIX A

TASK 1

Solve the following based on the pattern studied above.

No	Pattern Shape Input		Pattern Shape Output
1	⬬ΔΔ ⬬ΔΔΔΔΔ	=	
2	⬬ΔΔΔΔΔ⬬ΔΔΔΔ	=	
3	ΔΔΔ⬬ ΔΔΔΔΔ	=	
4	ΔΔΔΔ ⬬ ΔΔΔΔΔΔ	=	
5	Δ⬬ΔΔΔΔΔΔ	=	
6	⬬ΔΔΔΔΔΔ⬬ ΔΔΔΔ	=	
7	⬬ΔΔΔΔΔΔ ⬬ ΔΔΔ	=	
8	ΔΔΔ ⬬ ΔΔΔΔΔΔ	=	

TASK 2

Study the following examples. See if you can find a pattern.

No	If Pattern Shape Input is		Then Pattern Shape Output
1	⬬3 ⬬5	=	⬬8
2	3 ⬬ 5	=	⬬2

Solve the following based on the above.

No	IF PATTERN SHAPE INPUT IS		THEN PATTERN SHAPE OUTPUT
1	⬬3 ⬬ 5	=	
2	3 ⬬ 7	=	
3	⬬5 ⬬ 3	=	
4	⬬6 ⬬7	=	
5	6 ⬬ 8	=	
6	2 ⬬ 7	=	
7	1 ⬬ 6	=	
8	⬬3 ⬬7	=	

TASK 3

Study the following examples. See if you can find a pattern.

No	IF PATTERN SHAPE INPUT IS		THEN PATTERN SHAPE OUTPUT
1	⬬3 ⬬2 ⬬6	=	⬬ 11
2	3 ⬬7 ⬬4	=	⬬ 8

Solve the following based on the above.

No	IF PATTERN SHAPE INPUT IS		THEN PATTERN SHAPE OUTPUT
1	⬬3 ⬬5 ⬬4	=	
2	3 ⬬5 ⬬4	=	
3	⬬5 ⬬3 ⬬6	=	
4	⬬2 ⬬7 ⬬3	=	
5	6 ⬬8 ⬬7	=	
6	6 ⬬7 ⬬8	=	
7	1 ⬬6 ⬬6	=	
8	⬬3 ⬬7 ⬬6	=	